CSCE 643 Spring 2017

Introduction to Robot and Computer Vision on Multiple View Geometry

Project 2 Report

Question 1: H matrix computed using DLT

1. **Mathematic Induction**

A projective transformation from the first image frame to the second image frame is a homography relation. It can be define as $X' = HX$

$$\begin{bmatrix} x_1{}' \\ x_2{}' \\ x_3{}' \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

We know that the cross product $X' \times HX = 0$.

Let $X_i' = (x_i',\ y_i', w_i{}')$ and the j-th row of the matrix H is denoted by a vector h_j^T, the matrix form of the cross product can be written as

$$X' \times HX = \begin{bmatrix} y_i' h_3^T x_i - \ w_i' h_2^T x_i \\ w_i' h_1^T x_i - \ x_i' h_3^T x_i \\ x_i' h_2^T x_i - \ y_i' h_1^T x_i \end{bmatrix}$$

$A_i h = 0$ is an linear equation where h is the unknown. For $j = 1,2,3$, three equation can be obtained to solve h.

$$\begin{bmatrix} 0^T & -w_i' x_i^T & y_i' x_i^T \\ w_i' x_i^T & 0^T & -x_i' x_i^T \\ -y_i' x_i^T & x_i' x_i^T & 0^T \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = 0$$

Because the third row is the linear combination of the first two rows, only two equations are linearly independent in the matrix.

$$\begin{bmatrix} 0^T & -w_i' x_i^T & y_i' x_i^T \\ w_i' x_i^T & 0^T & -x_i' x_i^T \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = 0$$

Thus, A_i becomes a 2×9 matrix for each point correspondence. Four points correspondence are enough to get an exact solution for H. In the question, we are required to select 20 pairs of point correspondence, which make $A_i h = 0$ overdetermined. We have to use the Direct Linear Transformation (DLT) algorithm, and find the SVD of A. By SVD, we can find the unit singular vector that is corresponding to the smallest singular value, and the last column of V is the h vector. The homography matrix can be acquired from the h vector.

Result of Question 1:

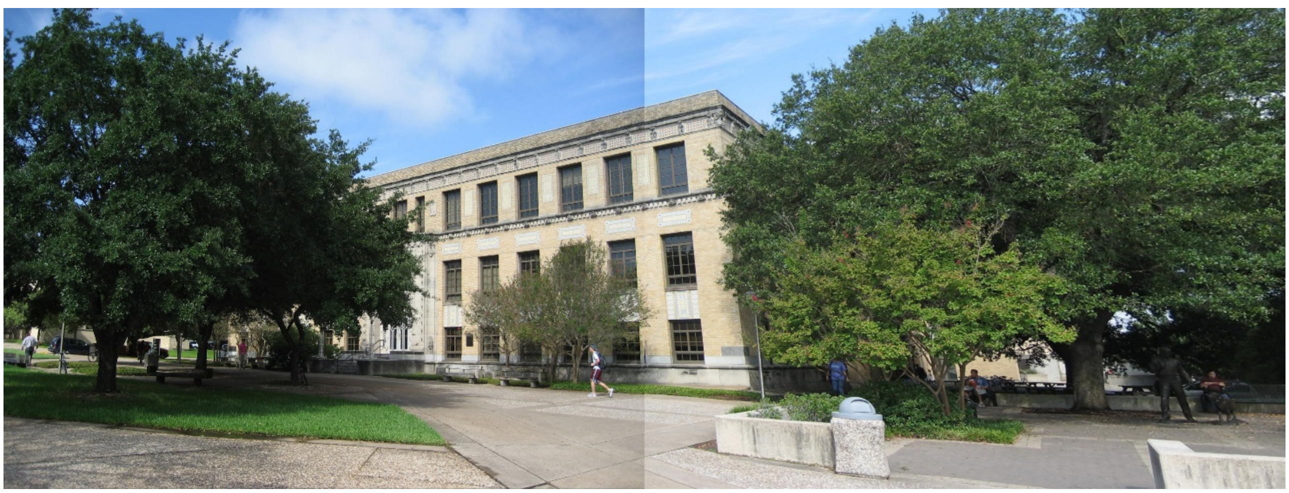
The image below shows the homography matrix I found. I also use the findHomography() in OpenCV to help verify the result. If the homography matrix H is divided by the h_{33} element, the result is close to the H matrix found by the findHomography().

```
Homography matrix H =
[-0.00030729276, -4.119374e-05, -0.99871045;
  0.0001709374, -0.00058062148, -0.050758194;
  1.5905927e-07, 1.8798175e-08, -0.00065356673]

Verify H by dividing h33
[0.4701781, 0.063029125, 1528.0925;
  -0.26154545, 0.88838899, 77.663368;
  -0.00024337112, -2.8762443e-05, 1]

Verify by OpenCV findHomography() =
[0.5028547214301783, 0.08679617067955796, 1524.43763377547;
  -0.2547443019096499, 0.9122624775444854, 67.94467496138512;
  -0.0002312742932471423, -1.680711406883824e-05, 1]
```

The panorama result of DLT:



Question 2: H matrix computed using nomalized DLT. In your report, please tell me how do you normalize the images.

In the normalized DLT algorithm, we have to map X_i & X_i' to another image coordinates by the transformation matrix T & T'.

$$T = \begin{bmatrix} s & 0 & t_x \\ 0 & s & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$\tilde{X} = TX$, $\tilde{X}' = T'X'$

Let $X = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$

$$\tilde{X} = TX = \begin{pmatrix} sx + t_x \\ sy + t_y \\ 1 \end{pmatrix} = \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ 1 \end{pmatrix}$$

(i) If we translated the points to the centroid of $\tilde{X}$ (the origin)

$$\tilde{X} = \begin{pmatrix} \bar{\tilde{x}} \\ \bar{\tilde{y}} \\ 1 \end{pmatrix} = \begin{pmatrix} s\bar{x} + t_x \\ s\bar{y} + t_y \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$\Rightarrow t_x = -s\bar{x}$, $t_y = -s\bar{y}$

(ii) The points are the scaled so that the average distance from the origin is equal to $\sqrt{2}$.

$$\Rightarrow \sqrt{2} = \text{Average distance} = \frac{1}{n} \sum_i \sqrt{\tilde{x}_i^2 + \tilde{y}_i^2}$$

$$= \frac{1}{n} \sum_i \sqrt{(sx_i - s\bar{x})^2 + (sy_i - s\bar{y})^2}$$

$$= \frac{s}{n} \sum_i \sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}$$

$$\Rightarrow \text{The scale } s = \frac{\sqrt{2}}{\frac{1}{n} \sum_i \sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}}$$

(iii)

$$\begin{array}{ccc} X & \xrightarrow{H} & X' \\ T \downarrow & & \downarrow T' \\ \tilde{X} & \xrightarrow{\tilde{H}} & \tilde{X}' \end{array} \quad \Rightarrow \quad H = (T')^{-1} \tilde{H} T$$

Result of Question 2:

The image below shows the homography matrix I found. I also use the findHomography() in OpenCV to help verify the result. If the homography matrix H is divided by the h_{33} element, the result is close to the H matrix found by the findHomography().

```
Homography matrix H =
[-0.57569975, -0.072891004, 0.0093085868;
  0.051846661, -0.57277238, 0.001791054;
  0.028799186, 0.0025569426, -0.5758251]

The denormalized Homography matrix H =
[-0.308938, -0.049078204, -943.32019;
  0.15792184, -0.55995315, -43.705658;
  0.00014400081, 1.2785146e-05, -0.61862892]

Verify H by dividing h33
[0.49939147, 0.079333834, 1524.8563;
  -0.25527716, 0.9051519, 70.649231;
  -0.00023277412, -2.0666906e-05, 1]

Verify by OpenCV findHomography() =
[0.5028547214301783, 0.08679617067955796, 1524.43763377547;
  -0.2547443019096499, 0.9122624775444854, 67.94467496138512;
  -0.0002312742932471423, -1.680711406883824e-05, 1]
```

The panorama result of the normalized DLT: