Question 1 (30%): Use the four point rectification method to rectify the image (a). (Page. 35 of the textbook).

Answer:

A projective transformation from the first view (image view, X) to the second view (world view, X') is defined as

$$X' = HX$$

$$\begin{bmatrix} x'_1 \\ x'_2 \\ x'_3 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Let a point (x, y) on the first view image coordinate defined as

$$x = \frac{x_1}{x_3}, y = \frac{x_2}{x_3}$$

Let a point (x', y')on the second view (world coordinate) defined as

$$x' = \frac{x'_1}{x'_3}, y' = \frac{x'_2}{x'_3}$$

We can find the following equation from the equation of the projective transformation.

$$x'_1 = h_{11}x_1 + h_{12}x_2 + h_{13}x_3 = \frac{1}{x_3}(h_{11}x + h_{12}y + h_{13})$$

$$x'_2 = h_{21}x_1 + h_{22}x_2 + h_{23}x_3 = \frac{1}{x_3}(h_{21}x + h_{22}y + h_{23})$$

$$x'_3 = h_{31}x_1 + h_{32}x_2 + h_{33}x_3 = \frac{1}{x_3}(h_{31}x + h_{32}y + h_{33})$$

Thus,

$$x' = \frac{x'_1}{x'_3} = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}}$$

⇒ Eq(1): $h_{11}x + h_{12}y + h_{13} - h_{31}xx' + h_{32}yx' + h_{33}x' = 0$

$$y' = \frac{x'_2}{x'_3} = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}}$$

⇒ Eq(2): $h_{21}x + h_{22}y + h_{23} - h_{31}xy' + h_{32}yy' + h_{33}y' = 0$

One point can provide two equation. We need four points to provide eight equations so that we can find the elements of H. We pick 4 points that can help remove perspective distortion. That is, the four points should be on the same plane and form a rectangle on the second view (world coordinate).

The transformation of 4 points are defined as

$$(x_{p0}, y_{p0}) \leftrightarrow (x'_{p0}, y'_{p0})$$
$$(x_{p1}, y_{p1}) \leftrightarrow (x'_{p1}, y'_{p1})$$
$$(x_{p2}, y_{p2}) \leftrightarrow (x'_{p2}, y'_{p2})$$
$$(x_{p3}, y_{p3}) \leftrightarrow (x'_{p3}, y'_{p3})$$

Thus, we can substitute the value of the 4 points to the Eq(1) and Eq(2) to create the following matrix.

$$\begin{bmatrix} x_{p0} & y_{p0} & 1 & 0 & 0 & 0 & -x_{p0}x'_{p0} & -y_{p0}x'_{p0} - x'_{p0} \\ 0 & 0 & 0 & x_{p0} & y_{p0} & 1 & -x_{p0}y'_{p0} & -y_{p0}y'_{p0} - y'_{p0} \\ x_{p1} & y_{p1} & 1 & 0 & 0 & 0 & -x_{p1}x'_{p1} & -y_{p1}x'_{p1} - x'_{p1} \\ 0 & 0 & 0 & x_{p1} & y_{p1} & 1 & -x_{p1}y'_{p1} & -y_{p1}y'_{p1} - y'_{p1} \\ x_{p2} & y_{p2} & 1 & 0 & 0 & 0 & -x_{p2}x'_{p2} & -y_{p2}x'_{p2} - x'_{p2} \\ 0 & 0 & 0 & x_{p2} & y_{p2} & 1 & -x_{p2}y'_{p2} & -y_{p2}y'_{p2} - y'_{p2} \\ x_{p3} & y_{p3} & 1 & 0 & 0 & 0 & -x_{p3}x'_{p3} & -y_{p3}x'_{p3} - x'_{p3} \\ 0 & 0 & 0 & x_{p3} & y_{p3} & 1 & -x_{p3}y'_{p3} & -y_{p3}y'_{p3} - y'_{p3} \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = 0$$

This can be simplified as $Ah = 0$

We can apply the singular value decomposition (SVD) to the matrix A, and h can be acquired from the null vector. Thus, we can gain the homography matrix H. The image without perspective distortion can be calculated by $X' = HX$.

Important matrices:

```
A =
[1012, 446, 1, 0, 0, 0, -506000, -223000, -500;
  0, 0, 0, 1012, 446, 1, -506000, -223000, -500;
  1298, 435, 1, 0, 0, 0, -778800, -261000, -600;
  0, 0, 0, 1298, 435, 1, -649000, -217500, -500;
  1295, 689, 1, 0, 0, 0, -777000, -413400, -600;
  0, 0, 0, 1295, 689, 1, -777000, -413400, -600;
  1015, 736, 1, 0, 0, 0, -507500, -368000, -500;
  0, 0, 0, 1015, 736, 1, -609000, -441600, -600]

H =
[-0.00011152254;
  5.4590499e-05;
  -0.60093886;
  0.00029115594;
  -0.00041437207;
  -0.79929221;
  6.1805036e-07;
  1.0047957e-07;
  -0.0020491856]

Verify A*H = 0
[5.1188124e-08;
  1.6947467e-08;
  3.0317125e-07;
  1.4474389e-07;
  2.6088122e-07;
  3.3193822e-07;
  1.1426732e-07;
  1.1795009e-07]

Homography matrix H =
[-0.00011152254, 5.4590499e-05, -0.60093886;
  0.00029115594, -0.00041437207, -0.79929221;
  6.1805036e-07, 1.0047957e-07, -0.0020491856]
```

Input image of Q1:

The selected points are:

P0 (1012, 446) ---------- P1 (1298, 435)

| |

| |

P3 (1015, 736) ---------- P2 = (1295, 689)

Rectified result of Q1:

Pros:

- Remove the perspective distortion in one step.

Cons:

- The four point rectification method requires users to pick the correct point. If users do not pick the right points, the result of rectification will be wrong.
- The noises are larger because users have to select 4 points manually.